

**SOLUTION M12P HW Section 5.1 Graphing Sine and Cosine Function**

1. Given the wave functions below, indicate the period, amplitude, general formula for all the x-intercepts, Y-intercept, domain and range

	<b>PERIOD:</b> Around 2.1 <b>Amplitude:</b> = 1 <b>General Formula X-intercepts:</b> $X = 1.05n$ , where 'n' is an integer <b>Y intercept:</b> (0,0) <b>Domain:</b> 'x' can be all real numbers <b>Range:</b> [-1,1] or $-1 \leq y \leq 1$
	<b>PERIOD:</b> Around 4.2 <b>Amplitude:</b> = 1 <b>General Formula X-intercepts:</b> $X = 2.1n$ where 'n' is an integer <b>Y intercept:</b> (0,0) <b>Domain:</b> 'x' can be all real numbers <b>Range:</b> [-1,1] or $-1 \leq y \leq 1$
	<b>PERIOD:</b> Around 12.5 to 12.7 units, these are just estimates to how much it takes to repeat a cycle.... <b>General Formula X-intercepts:</b> $X = 1 \pm 12.5n$ and $X = 5.2 \pm 12.5n$ <b>Y intercept:</b> (0,-1) <b>Domain:</b> 'x' can be all real numbers <b>Range:</b> [-3,1] or $-3 \leq y \leq 1$
	<b>PERIOD:</b> Around 26 units <b>Amplitude:</b> = 1.5 <b>General Formula X-intercepts:</b> $X = 5 \pm 26n$ $X = 12.5 \pm 26n$ <b>Y intercept:</b> (0,-1.75) <b>Domain:</b> 'x' can be all real numbers <b>Range:</b> [-2.5,0.5] or $-2.5 \leq y \leq 0.5$

When looking for the period, find a spot where it begins a cycle and where it ends. If that's not good enough, find where two or three cycles will end. Then divide by the number of cycles.

2. When looking at an unit circle, at what angles are the “X” coordinates equal to zero? At what angles are the “Y” coordinates equal to zero?

The “x’coordinates are equal to zero when the angles in standard position are at  $90^\circ, 270^\circ, 450^\circ, \dots$ . These are angles when the terminal arm is pointing UP or Down. A formula for all these angles are:  $\theta = 90^\circ \pm (180^\circ)n$

The “y’coordinates are equal to zero when the angles in standard position are at  $0^\circ, 180^\circ, 360^\circ, \dots$ . These are angles when the terminal arm is pointing LEFT or RIGHT. A formula for all these angles are:  $\theta = (180^\circ)n$

3. What does the sine function represent in an unit circle? When are graphing the sine function, what are actually graphing? Explain:

The sine function is graphing the relationship of the “height” [y-coordinate] of the point on the unit circle as it is rotating around the origin with angle  $\theta$ . The “Y” axis of a sine function is the height and the “x” axis is the angle of rotation.

4. What does the cosine function represent in an unit circle? When are graphing the cosine function, what are actually graphing? Explain:

The cosine function is graphing the relationship of the “width” [X-coordinate] of the point on the unit circle as it is rotating around the origin with angle  $\theta$ . The “Y” axis of a cosine function is the value of the ‘x’ coordinate and the “x” axis is the angle of rotation.

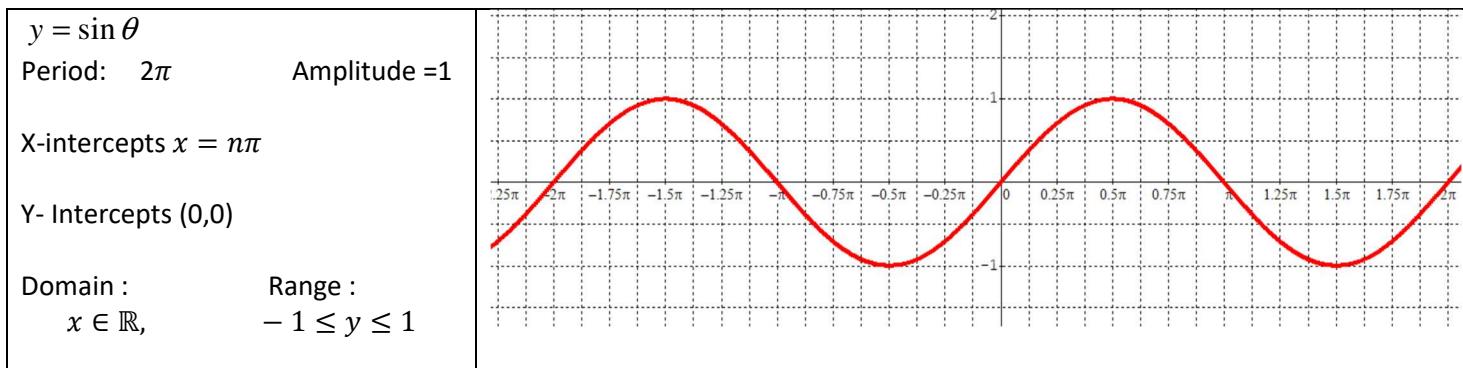
5. Why is the period of a Sine function and Cosine function both equal to  $2\pi$ ? Explain

It's because it takes  $360^\circ$  for to make a full rotation. OR in the case of radians, it takes  $2\pi$  to make a full rotation.

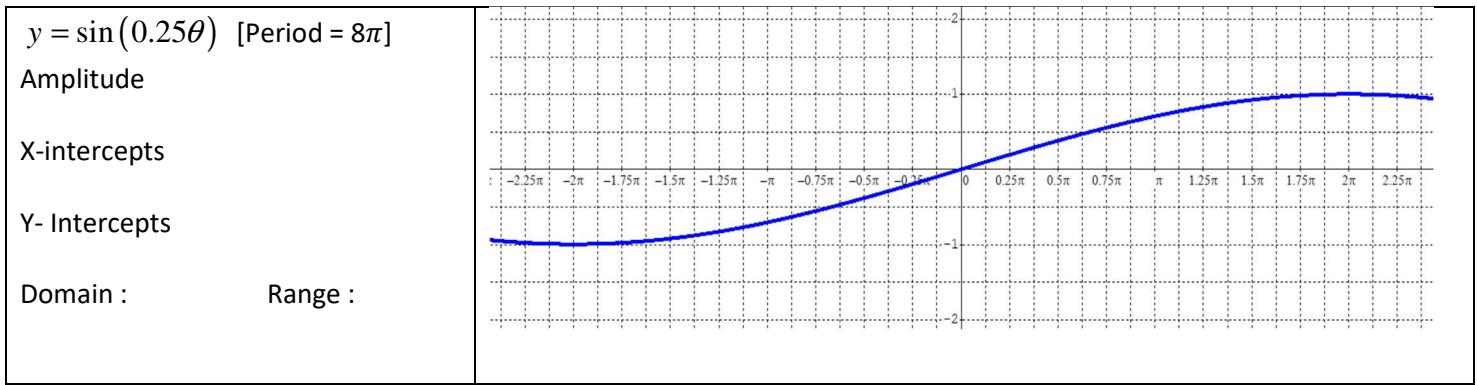
6. Why is the amplitude of a Sine Function and Cosine function both equal to 1? Explain:

Since the range of both the sine and cosine function is between -1 to 1, the amplitude will be half of the range.

7. Graph the following functions on the grid provided. Indicate the period, amplitude, “x” intercepts, “y” intercepts, domain, and range



$y = \cos \theta$ Period: $2\pi$ Amplitude = 1 X-intercepts $x = \frac{(2n+1)\pi}{2}$ Y- Intercepts (0,1) Domain : $x \in \mathbb{R}$ ,      Range : $-1 \leq y \leq 1$	
$y = -\sin \theta$ Period: $2\pi$ Amplitude = 1 X-intercepts $x = n\pi$ Y- Intercepts (0,0) Domain : $x \in \mathbb{R}$ ,      Range : $-1 \leq y \leq 1$	
$y = -\cos \theta$ Period: $2\pi$ Amplitude = 1 X-intercepts $x = \frac{(2n+1)\pi}{2}$ Y- Intercepts (0,1) Domain : $x \in \mathbb{R}$ ,      Range : $-1 \leq y \leq 1$	
$y = \sin(0.5\theta)$ [Period = $4\pi$ ] Amplitude X-intercepts Y- Intercepts Domain :      Range :	
$y = \cos(0.5\theta)$ [Period = $4\pi$ ] Amplitude X-intercepts Y- Intercepts Domain :      Range :	



8. When the graph of  $y = \sin x$  and  $y = \cos x$  are drawn on the same graph for  $0 < x < 2\pi$  in which quadrants do they intersect? What are the coordinates of the points of intersection?

The graphs of  $y = \sin x$  and  $y = \cos x$  intersect at points where  $\sin x = \cos x$  OR  $\tan x = 1$ . These points are where the angles are equal to  $45^\circ$  and  $225^\circ$ .

9. Given that  $\sin \theta > 0$  and  $\cos \theta < 0$ , what is the range of possible values of  $\theta$  if  $0 < \theta < 2\pi$ ?

If  $\sin x > 0$ , then the angle must be in quadrants 1 and 2. If  $\cos x < 0$ , then the angle must be in quadrants 2 and 3. The common quadrant between these two are in quadrant 2. Therefore, the angle is:  $\frac{\pi}{2} < \theta < \pi$ .

10. Indicate TRUE or FALSE:  $\sin \theta > 0$  and  $\cos \theta > 0$ , then  $\tan \theta$  can be either positive or negative.

FALSE, b/c the angle must be in Quadrant 1.

11. How many units should the graph of  $y = \sin x$  be shifted horizontally so that it will overlap the graph of  $y = \cos x$ ?

12. When the graph of  $y = \sin x$  and  $y = 0.5$  are drawn on the same graph for  $0 < x < 2\pi$  in which quadrants do they intersect? What are the coordinates of the points of intersection?

Finding the intersection points between  $y = \sin x$  and  $y = 0.5$  is the same as solving the equation:  $\sin x = 0.5$ . In the first cycle, the intersection points are in Q1 and Q2.  $x = \sin^{-1}(0.5)$ . Suppose we want the points on intersection in radians  $x = 30^\circ, 150^\circ$ .

and for all cycles, then we have:  $\left(\frac{\pi}{6} \pm n(2\pi), 0.5\right)$  and  $\left(\frac{5\pi}{6} \pm n(2\pi), 0.5\right)$

13. What is the amplitude and period of the graph  $y = A \sin(Bx)$  if  $A = -3$  and  $B = 2$ ?

Amplitude is 3 and period is  $\frac{2\pi}{B} = \pi$

14. If point "P" is on the unit circle with coordinates defined by  $(\sin \theta, \cos \theta)$ , what is  $\theta$  in standard position?

If the "x" coordinate becomes "sinx" and the "y" coordinate becomes cosx, this means that the "x" and "y" coordinates are equal. Therefore, the angles in standard position for these angles are:  $45^\circ$  and  $225^\circ$

15. If  $0^\circ \leq \theta \leq 180^\circ$  and  $\sin \theta \geq \cos \theta$ , then:

a)  $0 \leq \theta \leq 45^\circ$       b)  $45^\circ \leq \theta \leq 90^\circ$       c)  $45^\circ \leq \theta \leq 180^\circ$       d)  $90^\circ \leq \theta \leq 180^\circ$       e)  $0 \leq \theta \leq 90^\circ$

16. In  $\Delta ABC$ ,  $2 \cos B \cos A = \sin C$ . What kind of shape is the triangle?

a) Right triangle      b) Equilateral triangle      c) 45-45-90 triangle      d) Isosceles triangle

17.  $0 < \beta < 2\pi$  what does  $\beta$  need to be in order for  $\sin \beta > \cos \beta$  to be true?

A.  $\frac{\pi}{4} < \beta < \frac{\pi}{2}$  and  $\pi < \beta < \frac{5}{4}\pi$

B.  $\frac{\pi}{4} < \beta < \pi$

C.  $\frac{\pi}{4} < \beta < \frac{5}{4}\pi$

D.  $\frac{\pi}{4} < \beta < \pi$  and  $\frac{5}{4}\pi < \beta < \frac{3}{2}\pi$

18. A rectangle PQRS has side PQ on the x-axis and touches the graph of

$y = k \cos(x)$  at the point "S" and "R" as shown. If the length of  $PQ$  is  $\frac{\pi}{3}$

and the area of the rectangle is  $\frac{5\pi}{3}$ , what is the value of "k"?

